

**Exercise 2.2.10**

(Fixed points) For each of (a)–(e), find an equation  $\dot{x} = f(x)$  with the stated properties, or if there are no examples, explain why not. (In all cases, assume that  $f(x)$  is a smooth function.)

- a) Every real number is a fixed point.
- b) Every integer is a fixed point, and there are no others.
- c) There are precisely three fixed points, and all of them are stable.
- d) There are no fixed points.
- e) There are precisely 100 fixed points.

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**Solution**

Fixed points are values of  $x$  where  $\dot{x} = 0$ .

- a)  $\dot{x} = 0$
- b)  $\dot{x} = \sin \pi x$
- c) There are no examples because there can't be adjacent stable fixed points.
- d)  $\dot{x} = 1$
- e)  $\dot{x} = \prod_{n=1}^{100} (x - n)$ .